the large step discontinuities are appreciably in error. Errors of this magnitude render impossible the accurate computer design of complex microstrip matching networks for microwave power transistor amplifiers. On the other hand, with a set of equations approximating the junction model curves incorporated in a microstrip network optimization computer program, such amplifiers have been developed at the Naval Research Laboratory on a "work the first time" basis. The program directly generates corrected physical dimensions of the networks.

VI. CONCLUSIONS

No attempt is made to generalize the results presented herein. The equivalent circuit has been shown to be valid for large steps from 50-Ω lines, in the frequency range from 1 to 2 GHz, on 0.0635-cm-thick alumina substrate material. Other tests have indicated that the curves are usable for steps between any two impedances, neither of which is 50 Ω.

It should be recognized that great precision is required in the measurement of input impedances and physical dimensions, and in the knowledge of substrate dielectric constant, in producing the parameters for this junction model. This work shows that very large microstrip step effects can be accounted for, gives equivalent circuit parameters for a specific case, and shows the utility of the results.

It is hoped that the presentation of this information will stimulate further theoretical work on very large impedance steps in microstrip, so that more general analytical expressions can be developed to replace the specific results presented herein.

ACKNOWLEDGMENT

The authors wish to thank H. E. Heddings for his expert fabrication and measurement support.

REFERENCES

[1] A. A. Oliner, "Equivalent circuits for discontinuities in balanced strip transmission line," *IRE Trans. Microwave Theory Tech. (Special Symposium Issue on Microwave Strip Circuits)*, vol. MTT-3, pp. 134–143, Mar. 1955.
[2] A. Farrar and A. T. Adams, "Matrix methods for microstrip three-dimensional problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 497–504, Aug. 1972.
[3] V. Nalbandian and W. Steenaart, "Discontinuities in symmetric striplines due to impedance steps and their compensations," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 573–578, Sept. 1972.
[4] B. Aianco, M. Granar, and S. Ridella, "Comments on the existence of transmission zeros in microstrip discontinuities," *Alta. Freq.*, vol. XLI, Nov. 1972.
[5] P. Benedek and P. Silvester, "Equivalent capacities for microstrip gaps and steps," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 729–733, Nov. 1972.
[6] R. Horton, "Equivalent representation of an abrupt impedance step in microstrip line," *IEEE Trans. Microwave Theory Tech.* (Short Papers), vol. MTT-21, pp. 562–564, Aug. 1973.
[7] G. T. O'Reilly and R. E. Neidert, "Computer program for increasing the accuracy of impedance measurements performed with a Hewlett-Packard manual network analyzer," Naval Res. Lab., Washington, D. C., Memo. Rep. 2676, Nov. 1973.

Design of Partial Height Ferrite Waveguide Circulators

E. J. DENLINGER, MEMBER, IEEE

***Abstract*—This short paper presents a design procedure for the widely used three-port waveguide circulator that has a partial height ferrite post in the junction region. Design formulas and curves are derived for two configurations of partial height circulators: one has a short circuit at one end of the ferrite post, while the other has dielectric spacers at both ends. The design method was used to build two circulators for operation at 38 GHz and 60 GHz, respectively. Excellent agreement between theory and experiment was obtained in predicting the center frequency and required matching structure of these devices.**

Manuscript received December 26, 1973; revised March 11, 1974.
The author is with the RCA Corporation, Princeton, N. J. 08540.

I. INTRODUCTION

The most commonly used broad-band waveguide circulator incorporates a partial height ferrite post in the junction. However, due to the complexity of this configuration, no exact theory for designing such a circulator has been obtained. Theories for only the full height ferrite post have been attempted [1], [2]. These analyses involve matching the dominant mode fields in the connecting waveguides to a summation of fields due to modes within the junction and thus require elaborate computer programs. Owen and Barnes [3] proposed that the partial height ferrite junction circulator operates in a turnstile fashion with rotating modes propagating along the ferrite axis. Later on, Owen [4] measured the phase-frequency responses of the eigenvalues from which the principal field modes could be experimentally identified and built an X-band circulator based on these measurements. He could adjust the ferrite geometry to achieve a 120° separation of the eigenvalues over a broad frequency range. However, the instrumentation for these measurements involved multiple phase shifters, attenuators, couplers, a 3-way power divider, and an HP network analyzer. This equipment is not readily available for devices operating above X-band.

In this paper, approximate formulas are presented which simplify the design of partial height waveguide circulators. They apply to both the single-ended and double-ended configurations of the so-called compact turnstile device. The design is used to build 38- and 60-GHz circulators. Good correlation is achieved between measured and predicted performance.

II. THEORY

The two types of partial height ferrite circulators shown in Fig. 1(a) and (b) are the one-sided compact turnstile device, which has a short circuit at one end of the ferrite rod, while the other is a double-sided compact turnstile with dielectric spacers on both ends. With either type, two of the three eigen excitations propagate axially along the rod in rotating modes which are circularly polarized in opposite senses. Magnetizing the ferrite increases the propagation constant of one of these modes and decreases the propagation constant of the other since these modes experience different permeabilities. The third eigen-excitation does not couple into the ferrite rod along the symmetry axis. It simply uses the rod as a dielectric resonator with its center frequency and phase φ_3 a function of the effective dielectric constant of the dielectric–ferrite combination and the effective radius (including the effect of fringing fields) of the resonator. With the ferrite magnetized, a phase displacement of $2n\pi$

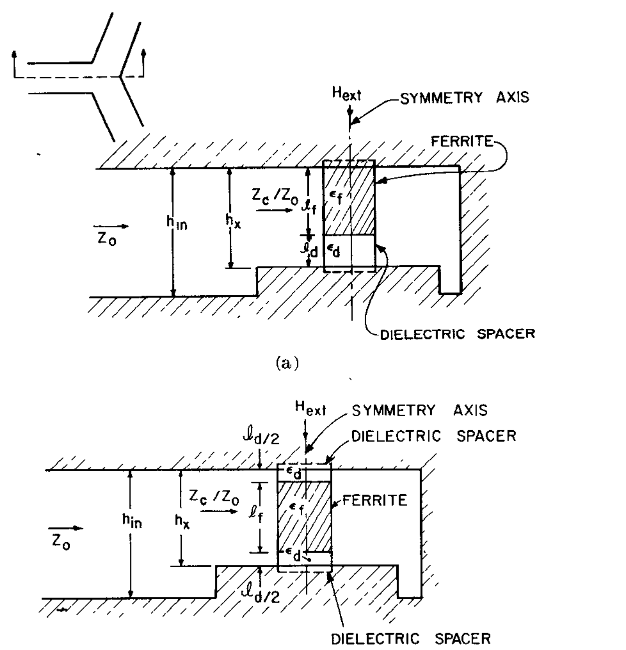

Fig. 1. (a) One-sided compact turnstile circulator. (b) Double-sided compact turnstile circulator.

is desired between the average phase of the rotating modes with phases φ_1 and φ_2 and the so-called in-phase dielectric resonator mode. Using the one-sided unit, the short circuit contributes a π-radian phase shift and the length and diameter of the ferrite rod is adjusted to give an additional average phase shift of $(2n - 1)\pi$ plus a phase advancement and retardation of $2\pi/3$ radians of the two rotating modes, respectively. With the double-sided unit, the rotating modes propagate only once through the rod. The π radians of phase shift are obtained by coupling modes into and out of the ferrite while the remaining required phase shift of $(2n - 1)\pi \pm 2\pi/3$ is obtained with a ferrite rod normally twice as long as that needed for the single-sided unit.

In order to obtain the proper phase relationship between the three eigen excitations, the propagation constants of the rotating modes must be known. The characteristic equation for a hybrid mode propagating along the axis of a longitudinally magnetized ferrite rod in air was derived in [5]. A simplified version good only for small values of κ/μ was used to calculate the propagation constants for the two HE_{11} rotating modes. The results are shown in Fig. 2. Fig. 2(a) shows one half the difference of the normalized propagation constants of the two modes as a function of ferrite rod diameter D_F and the off-diagonal permeability tensor component κ. Fig. 2(b) shows one half the sum of the propagation constants (average) as a function of the same variables. If the ferrite is saturated and the internal dc magnetic field is zero, κ is equal to m_s, the normalized saturation magnetization $\gamma 4\pi M_s/f$. The following relationships involving β_{ave}/β_0 and $\frac{1}{2}(\beta_{dif}/\beta_0)$ for the two types of partial height circulators must exist to achieve the $2\pi/3$ phase displacement between the rotating modes and the in-phase dielectric resonator mode:

$$2\beta_0 l_f \left(\frac{\beta_{ave}}{\beta_0}\right) = (2n-1)\pi$$

$$2\beta_0 l_f \left(\frac{1}{2}\frac{\beta_{dif}}{\beta_0}\right) = \frac{2\pi}{3} \qquad \text{for one-sided unit} \qquad (1)$$

$$\beta_0 l_f \left(\frac{\beta_{ave}}{\beta_0}\right) = (2n-1)\pi$$

$$\beta_0 l_f \left(\frac{1}{2}\frac{\beta_{dif}}{\beta_0}\right) = \frac{2\pi}{3} \qquad \text{for two-sided unit.} \qquad (2)$$

For either type, the normalized ferrite diameter D_f/λ_0 must be chosen in order to satisfy the equation

$$\frac{[\frac{1}{2}(\beta_{dif}/\beta_0)]}{(\beta_{ave}/\beta_0)} = \frac{2}{3(2n-1)}, \qquad \text{where } n = 1,2,3\cdots. \qquad (3)$$

Given the magnetization, $\kappa = m_s$, and the resonator mode number n, where the resonator modes are denoted by $HE_{1,1,n}$ for the double-sided unit and $HE_{1,1,(n-1/2)}$ for the one-sided unit, we can then find the required ferrite rod diameter by using Fig. 3 and the corresponding value of β_{ave}/β_0 from Fig. 2. The required ferrite rod length can then be calculated from either (1) or (2), depending upon which circulator type is chosen.

With regard to the choice of ferrite material, it is desirable to have as large a value of m_s as possible and yet avoid the region of high losses ($m_s \gtrsim 0.7$) since the difference in permeability of the two rotating modes ($\mu \pm \kappa$) should be large to maximize β_{dif}. This will allow operation of the device with a low resonator mode number and minimization of the path length difference between the in-phase and rotating modes. A large path length difference results in a narrow bandwidth for the circulator since the rate of phase change with frequency of the in-phase mode is much less than that of the rotating modes.

A low-loss dielectric is needed at the ends of the ferrite. The length of the dielectric section is selected to center the in-phase dielectric resonator mode at the proper frequency. The resonance condition is described by:

$$\frac{\pi D_{eff}}{\lambda_0}(\epsilon_{eff})^{1/2} = 1.84 \qquad (4)$$

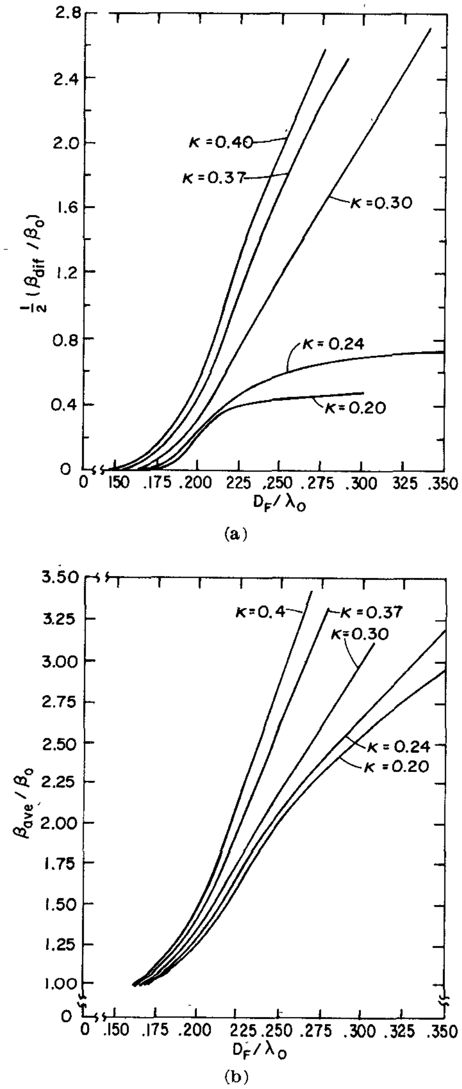

Fig. 2. One half the difference of the normalized propagation constants of the two HE_{11} modes versus the normalized ferrite rod diameter for different values of κ. (b) The average of the two HE_{11} mode normalized propagation constants versus the normalized ferrite rod diameter.

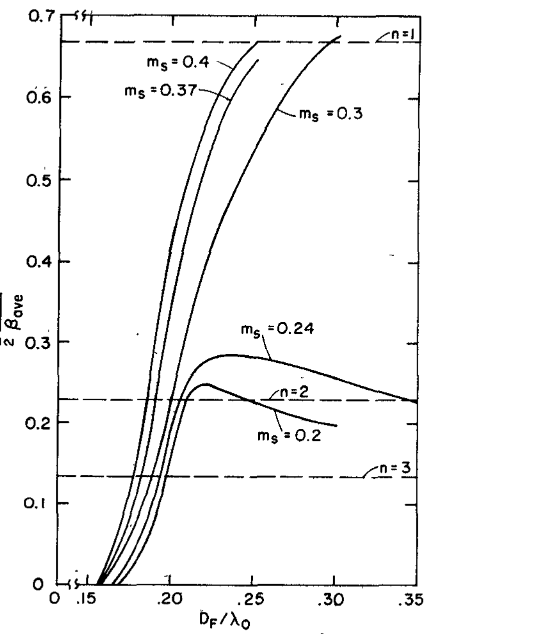

Fig. 3. One half the ratio of the difference and the average of the HE_{11} mode propagation constants versus the normalized ferrite rod diameter for different values of κ.

where $D_{eff} \cong 1.075 D_f$ to account for the fringing fields [6] (this is a rough approximation since in [6] the resonator was surrounded by dielectric) and ϵ_{eff} is the effective dielectric constant of the combined layers of ferrite and dielectric. For this dielectric resonator mode the electric field is perpendicular to the dielectric–ferrite interface and thus the total configuration can be approximately represented by series capacitors. Knowing ϵ_{eff} from (4) the dielectric length can then be determined by using the following equation:

$$\frac{l_f + l_d}{\epsilon_{eff}} = \frac{l_f}{\epsilon_f} + \frac{l_d}{\epsilon_d}$$

or

$$\frac{l_d}{l_f} = \frac{1 - (\epsilon_{eff}/\epsilon_f)}{(\epsilon_{eff}/\epsilon_d) - 1}. \quad (5)$$

The compact turnstile circulator admittance must include the admittances of the two counter-rotating modes propagating along the ferrite rod. As with the two resonator mode theory for stripline circulators, the total admittance can be written as

$$Y_T = Y^+ + Y^- = G\left(\frac{\kappa}{\mu}\right) + j\left[\left(\frac{\omega C_e}{2} - \frac{1}{2\omega L^+}\right) + \left(\frac{\omega C_e}{2} - \frac{1}{2\omega L^-}\right)\right] \quad (6)$$

where

$$L^\pm = L(\mu \pm \kappa) \quad (7)$$

$$L_e = \left(\frac{\mu^2 - \kappa^2}{\mu}\right) L = \mu_{eff} L \quad (8)$$

$$C_e = \epsilon_{eff} C$$

where ϵ_{eff} is the effective dielectric constant of the ferrite–dielectric combination in the junction area.

$$\omega_0 = \frac{1}{(L_e C_e)^{1/2}}, \quad \text{where } \omega_0 \text{ is the circulation frequency}$$

$$= \frac{\omega_0^+ + \omega_0^-}{2} \propto \beta_{ave}.$$

The requirement that one of the circulator ports be decoupled is met by making the phase angles between the split admittances ±30°. This gives

$$\frac{\omega_0 C_e}{2} - \frac{1}{2\omega_0 L^-} = -\frac{G(\kappa/\mu)}{2\sqrt{3}} \quad (9)$$

and

$$\frac{\omega_0 C_e}{2} - \frac{1}{2\omega_0 L^+} = +\frac{G(\kappa/\mu)}{2\sqrt{3}}. \quad (10)$$

By the use of (6)–(10), the following expression for the input conductance at band center and the loaded Q of the resonator can be obtained:

$$G\left(\frac{\kappa}{\mu}\right) = \sqrt{3}\left(\frac{\kappa}{\mu}\right)\left(\frac{C_e}{L_e}\right)^{1/2} \quad (11)$$

$$= \sqrt{3}\,\frac{\kappa}{\mu}\left(\frac{\epsilon_{eff}}{\mu_{eff}}\right)^{1/2}\left(\frac{C}{L}\right)^{1/2}$$

$$Q_L = \frac{n}{\sqrt{3}\kappa/\mu}. \quad (12)$$

where $(C/L)^{1/2}$ is taken to be the characteristic admittance of the circulator junction area adjacent to the ferrite post and n is the mode number of the resonance. Thus the input impedance at the ferrite–air interface is given by

$$Z_c/Z_0 = \frac{1}{\sqrt{3}}\left(\frac{\mu_{eff}}{\epsilon_{eff}}\right)^{1/2}\left(\frac{\mu}{\kappa}\right). \quad (13)$$

As mentioned previously and as evident from (12) for Q_L, the path length difference between the in-phase and rotating modes, or, i.e., the mode number of the resonator, is a big factor in determining the bandwidth of the compact turnstile circulator. Operation with the lowest possible mode number gives the best bandwidth. For broadband matching a quarter-wave transformer section can be used with waveguide height h_x being equal to that of the ferrite–dielectric post combination and the input waveguide height beyond the transformer determined from the following equation:

$$h_{in} = h_x/(Z_c/Z_0)^{1/2}. \quad (14)$$

Of course, another matching section must be used for conversion to standard size waveguide. The bandwidth obtained with quarter-wave transformer matching is estimated by using the following equation [7]:

$$\mathrm{BW} = \frac{S_{max} - 1}{S_{max}^{1/2} Q_L} = \frac{(S_{max} - 1)\sqrt{3}\kappa/\mu}{S_{max}^{1/2} n}. \quad (15)$$

Assuming that the ferrite is saturated with approximately zero internal dc magnetic field, the following expression can be used for estimating the required external dc magnetic field:

$$H_{ext} \cong N_Z(4\pi M_s). \quad (16)$$

The demagnetizing factor N_Z can be obtained from the graphs given in [8] for the general ellipsoidal-shaped ferrite.

III. EXPERIMENTAL RESULTS

Circulators were designed and built at 38 GHz and 60 GHz by using the equations described previously. Design parameters and experimental results are shown in Table I. For 38 GHz the configuration for broadest isolation bandwidth (16 percent) and lowest insertion loss (0.2–0.3 dB) was the double-sided unit operating in the lowest order mode $n = 1$. Nickel zinc ferrite with $4\pi M_s = 5000$ G was used along with thin layers of low-loss dielectric material having a dielectric constant of 2.35. At 60 GHz it was necessary to operate in a higher order mode $n = 2$ in order to obtain the required phase displacement between the in-phase and rotating modes. Also, in order to reduce losses, air was used as the dielectric spacer material.

TABLE I
CIRCULATOR DESIGN PARAMETERS ($\epsilon_f = 12.9$, $4\pi M_s = 5000$ G)

Parameter	Design for f_o = 38 GHz	Design for f_o = 60 GHz	
A. Design Parameters:		Option 1	Option 2
1. κ/μ	.368	0.234	0.234
2. D_f	.082	.040"	.069"
3. ℓ_f	.050	.109"	.047"
4. ϵ_{eff}	4.3	7.09	2.44
5. ϵ_d	2.35	1.0	1.0
6. ℓ_d/ℓ_f	.80	.074	.563
7. Z_c/Z_o	.70	0.902	1.54
8. $h_x = (\ell_f + \ell_d)$-(recess)[a]	.075	.112"	.068"
9. h_{in}	.090"	.118"	.055"
10. H_{ext} (oersted)	2400	850	2300
11. Q_L	1.57	4.96	4.96
12. Bandwidth[b]	11.6%	3.6%	3.6%
B. Experimental Results			
1. Bandwidth (20 dB Isolation)	16%	4%	5%
2. Insertion Loss	0.2-0.3 dB	.3-.6 dB	.2-.4 dB
3. Center Frequency (GHz)	37.75	60.5	60.0
4. H_{ext} (oersted)	2500	800	2400

[a] Recess is 0.005–0.010-in in each broadwall of the waveguide junction.
[b] The bandwidth was calculated for a maximum input VSWR of $S_{max} = 1.2$.

Thus the best circulator configuration for broadest bandwidth (due to minimum phase displacement of the modes) and lowest loss was the single-sided unit having a ferrite rod length of $3\lambda/4$. Because of the extremely nonlinear dependence of the propagation constants on the normalized ferrite diameter at 60 GHz ($m_s = 0.24$) there are two design options with widely different geometries and bias magnetic fields. The option having the longer and slimmer ferrite rod required significantly less bias field due to a smaller demagnetizing factor.

IV. CONCLUSIONS

A simple design procedure for the widely used partial height ferrite waveguide circulator has been formulated. It eliminates the need for sophisticated computer programs and/or elaborate experimental design techniques. Excellent agreement between theory and experiment has been obtained with circulators operating in the "turnstile" mode. Even greater isolation bandwidths can be achieved by stagger tuning with the standard disk resonator mode $TM_{\pm1,1,0}$ or the $TM_{0,1,\delta}$ resonance induced by a pin inserted along the axis of the ferrite rod. However, since design formulas for these configurations are much more difficult to obtain, experimental design techniques such as using an eigenvalue measuring set [4] must be used.

ACKNOWLEDGMENT

The author wishes to thank C. P. Wen for his helpful suggestions and H. Sobol for reading the manuscript. Assistance from N. Klein and H. Davis is also acknowledged.

REFERENCES

[1] J. B. Castillo, Jr., and L. E. Davis, "Computer-aided design of three-port waveguide junction circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 25–34, Jan. 1970.
[2] M. E. El-Shandivily, A. A. Kamal, and E. A. F. Abdallah, "General field theory treatment of *H*-plane waveguide junction circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 392–403, June 1973.
[3] B. Owen and C. E. Barnes, "The compact turnstile circulator," *IEEE Trans. Microwave Theory Tech. (1970 Symposium Issue)*, vol. MTT-18, pp. 1096–110, Dec. 1970.
[4] B. Owen, "The identification of modal resonances in ferrite loaded waveguide Y-junctions and their adjustment for circulation," *Bell Syst. Tech. J.*, vol. 51, pp. 595–627, Mar. 1972.
[5] W. H. Aulock and C. E. Fay, *Linear Ferrite Devices for Microwave Applications*. New York: Academic, 1968, ch. 3.
[6] D. Masse, "Broadband microstrip junction circulators," *Proc. IEEE*, vol. 56, pp. 352–353, Mar. 1968.
[7] L. K. Anderson, "An analysis of broadband circulators with external tuning elements," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-15, pp. 42–47, Jan. 1967.
[8] B. Lax and K. Button, *Microwave Ferrites and Ferrimagnetics*. New York: McGraw-Hill, 1962, pp. 160–162.

Letters

Guided Waves Along Graded Index Dielectric Rod

RYOZO YAMADA AND YASUNOBU INABE

Abstract—**By a modification of the Kurtz and Streifer procedure, the coupled second-order differential equations for the field components of guided modes along a graded index dielectric rod surrounded by a homogeneous medium were solved directly. Using the results, the eigenvalue equations which are consistent with those of the simple core-cladding-type dielectric fiber in the region near the cutoffs were obtained.**

Optical transmission through a graded index glass fiber has been given extensive attention. The theoretical study of guided waves in a focusing medium has been done by many investigators. Among them, Kurtz and Streifer have treated this problem on the basis of the circular cylindrical coordinates and solved approximately the linear homogeneous fourth-order differential equation, and applied the results to the wave propagation guided by an enclosed circular cylindrical graded index dielectric rod [1]–[3].

In this letter, we deal with the guided waves along a graded index dielectric rod such as Selfoc. By a modification of the Kurtz and Streifer procedure, we solve directly the coupled second-order differential equations approximately and apply the results to the guided waves along the rod. We adopt here the notations that have been defined in their paper [1].

We assume that the dielectric constant distribution is in the form

$$\epsilon = \epsilon_1\left(1 - \delta\left(\frac{r}{a}\right)^2\right), \qquad r < a \tag{1}$$

and

$$\epsilon = \epsilon_2, \qquad r > a \tag{2}$$

where

$$\epsilon_1 > \epsilon_2.$$

We express the axial components of electric and magnetic fields as

$$E_\zeta = E_{n\zeta} \exp[j(\omega t - \beta z)] \cos(n\phi + \theta) \tag{3}$$

$$H_\zeta = H_{n\zeta} \exp[j(\omega t - \beta z)] \sin(n\phi + \theta) \tag{4}$$

and further we write

$$E_{n\zeta} = [\epsilon_1(1-\chi)]^{-1/4}\phi \quad \text{and} \quad H_{n\zeta} = [\epsilon_1(1-\chi)]^{1/4}\psi/\eta_0 \tag{5}$$

where

$$\chi = 1 - \beta^2/k^2\epsilon_1 \qquad \eta_0 = (\mu_0/\epsilon_0)^{1/2}$$

instead of

$$\phi = \epsilon_1^{1/4}E_\zeta \qquad \psi = -i\epsilon_1^{-1/4}\eta_0 H_\zeta$$

in the paper [1].

We obtain the wave equations for ϕ and ψ in the rod:

$$\frac{d^2\phi}{dz^2} + \left[\frac{1}{z} + \frac{2z}{1-z^2} - 2\chi z\right]\frac{d\phi}{dz} + \left[b^2(1-z^2) - \frac{n^2}{z^2}\right]\phi = \frac{-2n\psi}{1-z^2} + 2n\chi\psi \tag{6}$$

$$\frac{d^2\psi}{dz^2} + \left[\frac{1}{z} + \frac{2z}{1-z^2}\right]\frac{d\psi}{dz} + \left[b^2(1-z^2) - \frac{n^2}{z^2}\right]\psi = \frac{-2n\phi}{1-z^2} \tag{7}$$

where

$$z = (\delta/\chi)^{1/2}(r/a) \qquad b^2 = (ka\chi)^2\epsilon_1/\delta.$$

Since χ is small for the guided modes whose fields are bounded near the region of maximum permittivity, we neglect the terms $2\chi z(d\phi/dz)$, $2n\chi\psi$, and reduce (6) and (7) to

$$\frac{d^2\phi}{dz^2} + \left[\frac{1}{z} + \frac{2z}{1-z^2}\right]\frac{d\phi}{dz} + \left[b^2(1-z^2) - \frac{n^2}{z^2}\right]\phi + \frac{2n\psi}{1-z^2} = 0 \tag{8}$$

Manuscript received January 14, 1974; revised March 13, 1974.
The authors are with the Department of Electronic Engineering, Shizuoka University, Jyohoku, Hamamatsu, Japan.